Topology Vulnerability Analysis of several Urban Metro Networks*

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ABSTRACT

In modern cities, urban metro systems gradually become an important transportation tool. The failure of metro may influence citizens' travel and cause economic losses. It is a focal problem that assessing the vulnerability of metro networks at home and abroad. Several metro networks are modeled by a modified Space L, in which metro interchange and travel time are involved. The properties of these metro networks are calculated at first, showing that at the same size, the average degree is larger, the network efficiency is better. Then the vulnerabilities of metro networks under random attack and three malicious attacks are studied and discussed. It is discovered that the metro networks are vulnerable to the biggest travel-time-efficiency node-based attack(EA) and the highest betweenness node-based attack(BA), and robust against random attack. The four attacks harm Tokyo metro network least, which has a big size, the max average degree and clustering coefficient of the seven metro networks. Finally, the top ten stations in order under EA and BA are respectively listed as a case study of Shanghai metro.

CCS CONCEPTS

• **Network → Network properties → Network structure →** Topology analysis and generation

KEYWORDS

metro network, vulnerability analysis, key station, complex network theory

1 INTRODUCTION

In recent years, with the continuous construction and improvement of urban metro network system, more and more people travel by subway because of its convenience, security and speediness. As an important fast transportation tool, urban metro system plays an irreplaceable role in daily life for public of modern cities. And small disturbances may lead to wild effects on metro system. The disturbances can be natural disasters, power failures, terrorist activities and so on. So, many researches on metro network have been done to reduce the topological vulnerability of metro network when perturbations affect metro network system.

Up to now, the vulnerability of transportation systems have been defined and studied by many experts and scholars. Berdica [1] proposed that the concept of vulnerability should be defined as the susceptibility to incidents that can result in a considerable reduction in network serviceability. And Åke J. Holmgren [2] defined the vulnerability of the system as the sensitivity to such threats and hazards. For decades, with the putting forward of the WS small-world network model [3-4] and the BA scale-free network model [5], complex network theory applied to the study on the vulnerability of the real metro networks has been highly appreciated and is underway at home and abroad now. Chuanfeng Han [6] studied how the performance of several subway networks in China is affected under different ways of the removal of vertices, including the intentional attacks and random attack. Daniel [7] conducted a comprehensive analysis of the vulnerability of urban rail transit networks, assessing the station vulnerability and network vulnerability of Shanghai Metro based on the proposed evaluation model. Hong Cai [8] analyzed the vulnerability of Beijing's metro network to discuss the effects of flow impact on the vulnerability of the metro network.

Lots of studies [9-12] have been made based on complex network theory in order to analyze the vulnerability of metro network. Most of them lacked systematical analyses of urban rail transit networks, which may hinder accurate the definition, design and performance evaluation of the system, while these are particularly useful for the public transportation planners and practitioners [13]. The aim of the study is to raise a comprehensive analytical method to quantitatively assess station vulnerabilities of metro networks and study how transport ability of different metro networks acts under random and intentional attacks, finally to get some advice for metro security. The article is organized as follows. In section 2, the model of the metro network is introduced and the basic topological properties of several metro networks are listed. In section 3, the vulnerability of Shanghai metro is analyzed under different attacks and the key stations as well as the relevant evaluation indexes are listed in order of removing at first. Then the values of vulnerability of

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several metro networks are calculated and compared when it is subjected to different attack protocols in order to find out how the network structure affects the vulnerability of metro network system. Finally, the results and some advice are summarized in section 4.

2 Model Descriptions

A database of 7 cities' urban subway systems in China and Japan comprised of Beijing, Shanghai, Tokyo, Guangzhou, Shenzhen, Tianjin and Hong Kong, are reported here. In this section, the models of seven metro networks based on complex network theory and graph theory are constructed and the basic network properties are listed.

It is obvious that passengers would like to choose the path of whom travel time is the shortest. To be reality, the length of edge between two nodes is defined as the travel time or transfer time from one node to the other. Particularly, the transfer stations are split into several nodes equivalent to the number of passing lines so that the transfer time can be included in the path length.

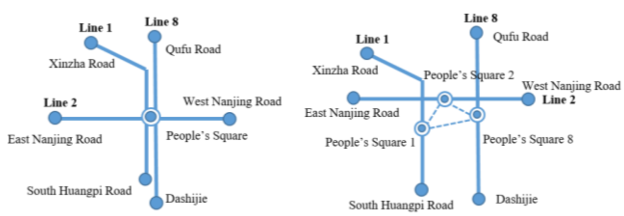

Figure 1: Spilt of transfer station.

Taking People's Square station of Shanghai metro as an example, as Figure 1 shows, the transfer station is divided into three virtual nodes, tagged with People's Square1, People's Square 2, People's Square8 respectively. And the three nodes are linked by the dashed edges in the Figure 1. The three lines are separated with only the linked three nodes, the length of the dashed edges is the corresponding transfer time.

In conclusion, of the study the metro network is regarded as an undirected graph, G = {V, E}, where the node set $V = \{v_i | i = 1, 2, \cdots\cdots, n\}$ (n denotes the number of nodes) represents metro stations, the edge set $E = \{e_{ij} | v_i, v_j \in V\}$ represents the direct connections between two nodes and the value of edges is the travel time or transfer time.

2.1 Preliminary Properties of real Metro Networks

Based on complex network theory, the average degree(<k>) is mean of the degree of all nodes in the network, defined as the number of edges directly connecting with node i [7]. To close the reality, the edge value from i to j are assigned as the travel time between two nodes. Refer to [14], the shortest path l_{ij} is defined as the least of all the travel time from node i to j. The average path length can be calculated by Equation (1). The network diameter(D) is written as (2). The clustering coefficient(C) is defined as (3), where k_i is the number of nodes directly connected to node i. Node betweenness (B_i) is defined as the number of shortest paths between any two nodes in the network passing node i, as shown in the equation (4). The travel-time-efficiency(E_t) in my study is the arithmetic mean of the reciprocal of shortest path length for all nodes, i.e. (5).

$$l_{ave} = \frac{1}{n(n-1)} \sum l_{ij} \tag{1}$$

$$D = \max_{1 \le i < j \le n} l_{ij} \tag{2}$$

$$C = \frac{1}{n} \sum_{i=1}^{n} \frac{E_i}{C_{k_i}^2} \tag{3}$$

$$B_i = \sum_{\substack{1 \le j < l \le n \\ j \ne i \ne l}} [n_{jl}(i)/n_{jl}] \tag{4}$$

$$E_t = \frac{1}{n(n-1)} \sum_{i \ne j} \frac{1}{l_{ij}} \tag{5}$$

Some preliminary properties of these metro networks above are calculated and as follows.

Table 1: Preliminary properties of metro networks

Network	n	l	<k>	C	The average path length	D	E_t
Beijing	339	380	2.24	0.0040	42.86	140.95	0.036
Shanghai	366	425	2.32	0.016	40.34	143	0.038
Tokyo	291	401	2.76	0.060	26.19	75	0.052
Shenzhen	198	227	2.29	0.013	32.92	125	0.049
Guangzhou	194	211	2.17	0.003	37.021	122	0.043
Hong Kong	114	131	2.29	0.038	30.33	83	0.069
Tianjin	113	117	2.07	0.0060	34.82	103	0.048

As Table 1 shows, <k> and C of Tokyo metro network are the largest, whereas its L and D are the smallest in the seven real metro networks. E of Hong Kong and Tokyo are similar and the biggest, when Beijing's is the minimum. The seven metro networks are divided into three groups according to size depend on the value of n: Beijing, Shanghai and Tokyo; Shenzhen, and Guangzhou; Tianjin and Hong Kong. Then it is found that the larger the clustering coefficient/average degree is, the better the travel-time-efficiency is. The two preliminary properties, the clustering coefficient and the average degree, are critical for network efficiency.

2.2 Attack strategies

As more vertices are removed, the network structure changes, leading to the distributions of the degree and the betweenness different from the initial ones [6]. It is quite clear that metro networks are always more vulnerable to the recalculated attacks than the initial ones. In this study, focusing on the recalculated attacks, we take into consideration four attack strategies: random attack(RA)、the largest degree node-based attack(DA)、the highest betweenness node-based attack(BA) and the largest travel-time-efficiency node-based attack(EA). Under the three malicious attacks, the most important node of the damaged network is removed based on the recalculated degree/betweenness/travel-time-efficiency distributions respectively at every step.

3 RESULTS AND DISCUSSION

Under attacks, the change of the travel-time-efficiency reflects the topological loss of the network more, and the change of the connected OD mainly reflects the functionality loss. To comprehensively analyze and quantitatively assess the vulnerability of metro networks, transport ability loss is introduced to measure the impact of attacks, defined as the difference of network connectivity between the initial and the damaged scenario. And transport ability loss is written as:

$$\text{transport ability loss} = 1/2\ \left(\frac{E_t(I)\text{-}E_t(D)}{E_t(I)} + \frac{P(I)\text{-}P(D)}{P(I)} \right)$$

where

E_t (G): denotes the travel-time-efficiency in the initial scenario, I.

E_t (D): denotes the travel-time-efficiency in the damaged scenario, D.

P(D): denotes the connected OD numbers in the initial scenario, I.

P(G): denotes the remaining connected OD numbers in the damaged scenario, D.

3.1 Vulnerability Analysis of several Metro Networks

The metro networks of seven cities recommended are studied in section 2 under four attacks as above. Figure 2 depicts the changes in the network loss of seven metro networks under random attack and intentional attacks. From the four diagrams it is clear that the variation trend of the network loss of seven metro networks are almost the same, the values of transport ability loss are alike with some small variations considering random attack and malicious attack. As can be seen from graph(a), the curves in the graph have view turns and are less smooth than the EA curves in graph(b). When 7.5% nodes were attacked, the transport ability loss of EA and BA are around 80% and then the loss curves tend to be gentle gradually. At the time, the networks are almost paralyzed, showing metro networks are vulnerable to EA and BA. When removing 7.5% nodes by DA and RA, the transport ability losses separately are around 50% and 40%, finding that networks are robust to RA and DA. Metro networks are always vulnerable to malicious attack, and robust against random attack.

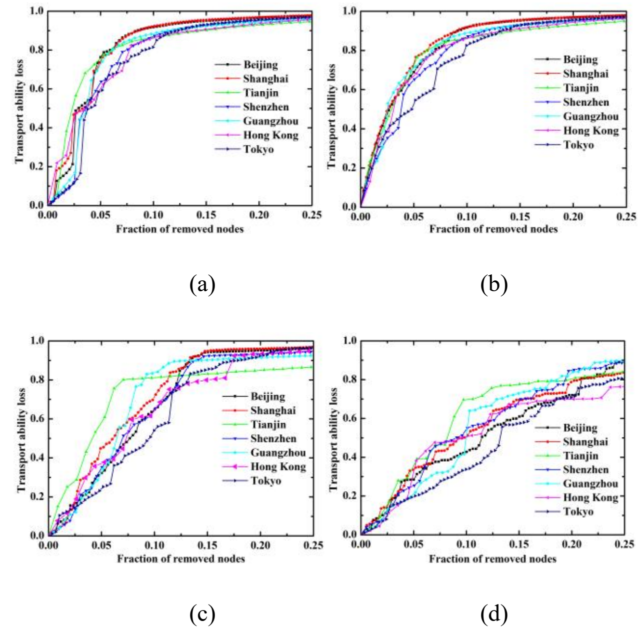

Figure 2: The change trend of transport ability loss of seven metro network under four attacks:(a)BA;(b)EA;(c)RA;(d)DA..

The highest betweenness node-based attack(BA) and the largest travel-time-efficiency node-based attack(EA) are more harmful because they remove the key nodes of network every time. Node betweenness is better to reflect node importance in network transport ability than node degree. The important difference between DA and BA is that the former concentrates on reducing the total number of edges in the network as fast as possible whereas the latter concentrates on destroying as many shortest path as possible [15]. And the study shows that destroying the shortest path can harm the network more.

Considering the seven networks, four attacks all harm Tokyo metro network least and Tianjin metro most as Figure 1 shows. It may because that Tokyo metro has a better size and network structure. And from Table 1, we can see that the average degree and average clustering coefficient of Tokyo metro network are both maximum of all seven networks. On the contrary, Tianjin metro network has the minimum. The average degree and average clustering coefficient can make sense to network structure.

3.2 Key stations: a case study of Shanghai metro

From last part, it's known that EA and BA make the metro network damage most. We want to study the two attack modes to get more useful conclusions. Table 2 lists the ten stations in the order and the corresponding transport ability loss of Shanghai metro network under EA and BA.

As we can see from Table 2, when attacking 2.73% nodes (10 nodes), the transport ability loss of Shanghai metro under EA and BA separately increase to 51.52% and 47.68%, which would have a strong impact on network transport ability of Shanghai metro. It is clear that the two group of stations under different attacks are completely different. And BA concentrates on destroying as many shortest path as possible, nevertheless EA concentrates on attacking the key nodes whose failure would influence travel-time-efficiency most. The two group of stations both are key/important nodes of Shanghai metro network and should be guarded specially in daily operation.

Table 2: Key stations and the transport ability loss under EA and BA

Order	Fraction of removed nodes	biggest travel-time-efficiency node-based attack(EA)		highest betweenness node-based attack(BA)	
		Station	Transport ability loss(ΔT_e)	Station	Transport ability loss(ΔT_e)
1	0.0027	Caoyang Road 11	0.0869	Century Avenue 2	0.0172
2	0.0055	Zhenping Road 7	0.1533	West Gaoke Road 7	0.0339
3	0.0082	Yishan Road 9	0.2135	Oriental Sports Center 11	0.1805
4	0.011	Shanghai Railway Station 1	0.2646	Hanzhong road12	0.1916
5	0.014	Shanghai South Railway Station 1	0.3196	Baoshan Road 3	0.2038
6	0.016	Longyang Road 2	0.3641	People's Square 8	0.2180
7	0.019	Hongkou Football Stadium 3	0.4103	East Nanjing Road 10	0.2414
8	0.022	Siping road 10	0.4414	Century Avenue 4	0.2706
9	0.025	Qufu Road 8	0.4899	Century Avenue 6	0.4674
10	0.027	Oriental Sports Center 8	0.5152	Jiaotong University 11	0.4768

4 CONCLUSIONS

This paper provides a comprehensive analytical method to quantitatively assess the network and station vulnerabilities of metro networks. Complex network theory is adopted to model the metro network and analyze properties refer to network structure. Special characteristics of metro networks, such as metro interchange and travel time, were taken into consideration. Transport ability loss considering the connected OD ratio and travel-time-efficiency is introduced to quantify the vulnerability of metro network under attacks.

A database about seven metro systems in China and Japan were collected firstly. Counting the basic properties of seven networks listed in Table1, it is found that of the same size the average degree is larger, the travel-time-efficiency is better. The transport ability loss of these seven networks under four attacks are respectively calculated and the corresponding graphs are depicted as Figure 2. And from the study the seven metro networks are vulnerable to the biggest travel-time-efficiency node-based attack and the highest betweenness node-based attack, and robust against random attack, demonstrating the conclusion generally applies to the most metro systems. And the change trend of transport ability loss of the networks under four attacks are similar because metro network owns its unique forming patterns different from networks of other fields. In addition, all the four attacks harm Tokyo metro network least, which has the max average degree and clustering coefficient of the seven metro networks. And the top ten stations in the order when attacking Shanghai metro network by EA and BA are both listed in Table 2 as a case study.

More attention and protect should be given to the two group of key stations. With the development of metro systems, metro networks will have a bigger size, more nodes and edges, and be more complex, making new key stations. The key stations should be acquired dynamically by calculating the vulnerabilities under EA and BA as in the study.

To improve the accuracy of the transport ability loss, passenger flow at different time and probabilities of node-failure should be considered in the future study. And how to protect the key stations should be also studied in future.

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